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Self-calibration for lensless color microscopy

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Lensless color microscopy (also called in-line digital color holography) is a recent quantitative 3D imaging method used in several areas including biomedical imaging and microfluidics. By targeting cost effective and compact designs, the wavelength of the low-end sources used is only imprecisely known, in particular because of their temperature and power supply voltage dependence. This imprecision is the source of biases during the reconstruction step. An additional source of error is the crosstalk phenomenon, *i.e.*, the mixture in color sensors of signals originating from different color channels. We propose to use a parametric inverse problem approach to achieve the self-calibration of a digital color holographic setup. This process provides an estimation of the central wavelengths and of the crosstalk. We show that taking the crosstalk phenomenon into account in the reconstruction step improves its accuracy. © 2017 Optical Society of America

OCIS codes: (090.1995) Digital holography; (090.1705) Color holography; (100.3.190) Inverse problems; (150.1488) Calibration; (100.3010) Image Reconstruction techniques.

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1. INTRODUCTION

Lensless microscopy is an established method to locate and estimate the size of micrometric objects in a volume [1, 2]. This technique is used in many areas including the study of fluid flows [3, 4] and biomedical imaging [5, 6]. Its main advantages are its robustness, simplicity and low cost implementation. Its main limitations are well known and inverse problem approaches can push back some of these limits [7, 8]. Lensless color microscopy applies to imaging of colored 3D and / or dephasing objects [9].

The use of CMOS sensors equipped with a Bayer filter and of LED or laser diodes makes it possible to obtain compact and cost-effective setups. However, uncertainty about the actual wavelengths of the laser sources may be a source of error during the holographic reconstruction step. In addition, the crosstalk phenomenon causes a mixture between the signals received on each channel.

In Section 2, we describe the principle of lensless color microscopy and we detail the possible causes and consequences of its incorrect calibration in the context of cost-effective setups. In Section 3, we describe a self-calibration method based on a reconstruction algorithm that uses a parametric inverse problem approach. In Section 4, we present a fast, simple and cost-effective experimental method to estimate the laser diode wavelengths and the mixing matrix of the crosstalk. Finally, in Section 5, we discuss and compare the theoretical precision limits of monochromatic and color holography.

2. LENSLESS COLOR MICROSCOPY AND ITS CALIBRATION ISSUES

A. Lensless color microscopy

The holographic setup used in this study is an in-line and lensless setup, as originally proposed by Dennis Gabor [10, 11], with a digital color sensor and three coherent lasers for illumination. The waves diffracted by the objects under study are recorded by the sensor. Figure 1 gives a schematic representation of the experimental setup.

We will illustrate our self-calibration method on our experimental setup in which three laser diodes are used, one red (*Thorlabs*TM, CPS635R, 1.5 mW), one green (*Thorlabs*TM, CPS532, 4.5 mW) and one blue (*Thorlabs*TM, CPS405, 4.5 mW). Their nominal wavelengths provided by the manufacturer are respectively: $\lambda_R^{nom} = 635$ nm, $\lambda_G^{nom} = 532$ nm and $\lambda_B^{nom} = 405$ nm. The holograms are recorded by a 12-bit CMOS sensor (*Basler*TM, daA2500-14uc) with $A \times B = 1944 \times 2592$ pixels with a $2.2 \mu\text{m}$ pitch.

three estimated values $\{z_R^{est}, z_G^{est}, z_B^{est}\}$ of the z_p distance obtained from three holograms recorded with the wavelengths λ^{real} and reconstructed assuming the wavelengths λ^{nom} satisfy the following system of equations:

$$\begin{cases} \lambda_R^{nom} \cdot z_R^{est} = \lambda_R^{real} \cdot z_p, \\ \lambda_G^{nom} \cdot z_G^{est} = \lambda_G^{real} \cdot z_p, \\ \lambda_B^{nom} \cdot z_B^{est} = \lambda_B^{real} \cdot z_p. \end{cases} \quad (2)$$

Rather than using the nominal values given by the manufacturer, these wavelengths should be calibrated, either using a spectrometer during a calibration step, or a self-calibration approach, as described in section B.

B.2. Calibration issues related to the CMOS color sensor

In a cost-effective perspective, color sensors using CMOS technology and a Bayer filter are a preferred choice. In this part, we describe the different crosstalk phenomena which occur on this type of sensors as well as their possible consequences during the holographic reconstruction.

The term "crosstalk" refers to the undesirable effects that occur when a signal transmitted on a channel modifies or impacts signals transmitted on the other channels. This global phenomenon leads to a reduction in sensitivity, poor separation of colors and a degradation of spatial and frequency resolution. The main crosstalk phenomena appearing on color sensors are spectral, optical and electronic [19–21]. We give below a brief description of each of them, focusing on how to quantify and reduce them.

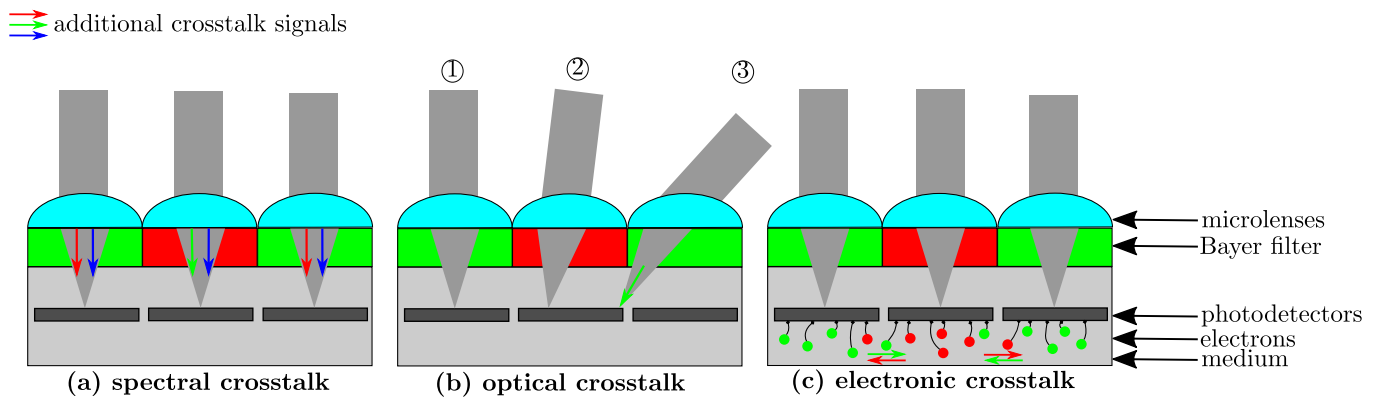


Fig. 2. Illustration of spectral (a), optical (b) and electronic (c) crosstalk phenomena.

Spectral crosstalk

CMOS sensors classically used in a cost-effective lensless color microscopy scheme often do not allow a trichromatic information to be obtained at each photodetector [22, 23]. Consequently, photodetectors are covered by a matrix of chromatic filters called "Bayer color filter array" [24], which filters the incident light by transmitting only one of the three primary components at each pixel. This step, called "mosaicking", is mandatory to record color images with this type of sensor, since the photodetectors used are sensitive to incident wave intensity over a wide spectral range. Spectral crosstalk is due to the Bayer filter covering the photodetector layer of the CMOS color sensors: their spectral responses are not ideal bandpass [25]. Figure 3 shows the spectral responses as a function of the wavelength for the filters of the *Basler*TM daA2500-14uc camera. A significant overlap between the different spectral responses can be noted.

In 2002, Yamagushi *et al.* presented one of the first use of digital sensors equipped with a Bayer filter for a color holography application [26]. The spectral crosstalk was mentioned in the sensor spectral responses but its characterization was not addressed. It should also be noted that a simple average of the received light intensity on each channel during a monochromatic exposure does not provide an accurate estimate of the spectral crosstalk. Indeed, the electrical signal produced by a low cost camera is not exactly linearly related to the intensity of the optical signal on the sensor (for example, an offset may be present). In addition, time-varying fringes can be observed on the hologram backgrounds obtained with low cost cameras. These fringes may be produced by a parallelism defect of the fine glass slides that protect the detector. This phenomenon makes it difficult to characterize the spectral crosstalk since the number of dark fringes does not remain constant during the recording, whereas the spectral crosstalk phenomenon remains constant. As a consequence, the gold standard method for quantitatively studying the spectral crosstalk phenomenon remains the establishment of the spectral responses of the Bayer filters. For example, Ozcan's group at UCLA has recently proposed to use a setup composed of a broadband laser source coupled to an acousto-optical filter that can be adjusted in wavelength around a fine bandwidth in order to accurately measure the spectral response of the filters [27].

In order to reduce the influence of spectral crosstalk, it is possible to use more selective pigments to stain Bayer filters such as those developed by *Micron Technologies*TM, Aptina. It is also possible to reduce the visual degradation induced by spectral crosstalk with additional numerical post-treatments and/or with a configuration of the color filter array different from that proposed by Bayer [28, 29]. However, in digital holography, low-pass filtering induced by demosaicking attenuates the high frequencies and thus blurs the reconstruction. In addition, it has been recently proposed to replace the traditional color filter array by a plasmonic color filter

directly lithographed close to the photodetectors [30–32]. This method has the advantage of simultaneously reducing the spectral and optical crosstalk while increasing the transmission of the filters. This method is not yet applicable in a low-cost scheme, though.

We propose in Section 4 a method for estimating the signals mixing induced by this crosstalk phenomenon and to take it into account for applications in lensless color microscopy.

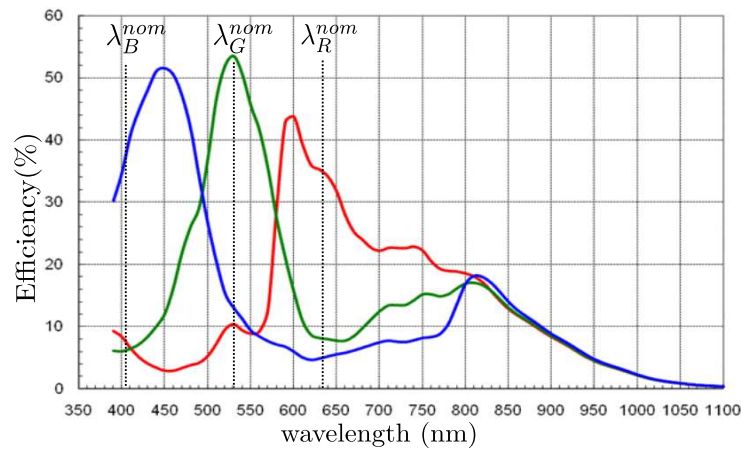


Fig. 3. Spectral responses of Bayer filter (according to Basler™, daA2500-14uc).

Optical crosstalk

Optical crosstalk occurs when the incident angle becomes too large for the microlenses array to focus light on the photodetector immediately below. In this case, a portion of the signal of interest may be lost in the gap that separates two adjacent photodetectors or, worse, impact a neighboring photodetector. Figure 2 illustrates this phenomenon. No optical crosstalk occurs at null or weak incidence angles (cases (b) ① and (b) ②). At higher incidence angles (case (b) ③), optical crosstalk can occur.

Since it is difficult to study this effect without taking into account the influence of other crosstalk phenomena, its importance is mainly determined by numerical simulations [33]. Since the incidence angle of the light beams striking the pixels located at the edge of the sensor is naturally higher than on the central part of the sensor, it is possible to optimize the placement of the microlenses with respect to the Bayer filters (by slightly shifting their center), in order to favor the convergence of the rays on the photodetectors of interest and thus reduce the optical crosstalk [34]. It is also possible to apply surface treatments (antireflective etc.) to the surface of photodetectors [35].

In lensless microscopy, local optical crosstalk can be disregarded since the laser is collimated and thus the incidence angles of the light on the sensor are small (less than 2°).

Electronic crosstalk

Electronic crosstalk is due to an interaction between signals received at two neighboring photodetectors. Some electrons created as a result of the detection of light in the depletion zone are diffused to adjacent photodetectors. This results in an additional error in the signal of interest as illustrated in Figure 2 (case (c)).

This phenomenon is commonly quantified by the use of an S-cube system to illuminate a single pixel (at normal incidence) and thus measure the proportion of signal leaking to the neighboring pixels due to the electronic crosstalk effect [20].

Since the advent of CMOS sensors, many improvements and optimizations of the architecture have been made by foundries to limit the effects of electronic crosstalk. It can be considerably reduced by adding an insulating cage around neighboring photodetectors which acts as a waveguide [35–38]. This technique also reduces the optical crosstalk [38]. It is also possible to increase the depth of the substrate [35]. The possible effects of this phenomenon are also taken into account in the overall crosstalk estimation scheme presented in Section 4.

Table 2 summarizes the causes, and the conventional means of estimating and correcting the various crosstalk phenomena.

Table 2. Spectral, optical and electronic crosstalk characteristics

Crosstalk:	spectral	optical	electronic
Cause:	Bayer filters	microlenses	electrons leakage
Classical method of quantification:	Bayer filters responses	numerical simulations	S-cube system
Classical methods of reduction:	filters improvement, post-treatments	microlenses position, photodetector treatment	waveguide, deep substrate

Section 4 demonstrates that an inaccuracy on wavelengths of low cost laser diodes as well as the crosstalk phenomenon can

significantly degrade the performance of the holographic reconstruction. We propose in Section 4 a simple, fast and cost effective experimental method to reduce the uncertainty on the wavelength of the sources and to characterize the crosstalk phenomenon.

3. RECONSTRUCTION USING A PARAMETRIC INVERSE PROBLEM APPROACH

Based on an adequate image formation model, parametric inverse problems approaches provide accurate reconstructions that are “optimal” in a signal processing point of view, under an additive white Gaussian noise hypothesis. They have been successfully used for metrological applications [39–42] and appear to be the best methods to calibrate a monochromatic holographic setup [43]. We propose to use such an approach to reconstruct a spherical opaque object and thus obtain an accurate calibration of the setup (*i.e.* low standard-deviation and bias). The proposed method can be considered cost-effective as it only requires an opaque sphere in addition to the lensless color microscope.

The basics and mathematical formulation of this technique are described below. We consider an opaque sphere characterized by the parameter vector $\theta = \{x_p, y_p, z_p, r_p\}$ in which $\{x_p, y_p\}$ represent the longitudinal coordinates in the object plane, z_p is the depth coordinate and r_p is the radius of the particle (see Figure 1).

The intensity diffracted by the object and recorded at the $\{x, y\}$ position in the sensor plane on channel c ($c \in \{R = 1, G = 2, B = 3\}$) is:

$$m_{\theta}^c(x, y) = m_0^c - \alpha_{\lambda(c)}^c \cdot g_{\lambda(c), \theta}(x, y) \quad (3)$$

in which m_0^c is an offset representing the intensity of the illumination wave, $\alpha_{\lambda(c)}^c$ is the amplitude factor of the interference pattern and $g_{\lambda(c), \theta}$ represents the analytical model of the diffraction pattern for the object characterized by the parameters set θ and illuminated by a plane wave at wavelength $\lambda(c)$.

Since the opaque sphere is illuminated by a collimated coherent source and z_p is chosen so that $z_p \gg \frac{4\pi \cdot r_p^2}{\lambda(3)}$, $g_{\lambda(c), \theta}$ can be described by a simplified Thompson’s model [44] in which the second order interference terms are neglected:

$$g_{\lambda(c), \theta}(x, y) = \frac{r_p}{2 \cdot \rho(x, y)} \cdot J_1 \left(\frac{2\pi \cdot r_p \cdot \rho(x, y)}{\lambda(c) \cdot z_p} \right) \cdot \sin \left(\frac{\pi \cdot \rho(x, y)^2}{\lambda(c) \cdot z_p} \right) \quad (4)$$

where $\rho(x, y) = \sqrt{(x - x_p)^2 + (y - y_p)^2}$ and J_1 is the first order Bessel function of the first kind.

Therefore, the color hologram model $m_{\theta}(x, y)$ at position (x, y) on the sensor plane is the vector given in equation 5 in which each component is associated with one of the three sensor channels c .

$$m_{\theta}(x, y) = \begin{pmatrix} m_{\theta}^R(x, y) \\ m_{\theta}^G(x, y) \\ m_{\theta}^B(x, y) \end{pmatrix} \quad (5)$$

The inverse approach consists of finding the best set of parameters θ from the recorded color hologram d , *i.e.* the set θ that minimizes, in the least squares sense, the distance to the data:

$$C(\theta) = \sum_{c \in \{R=1, G=2, B=3\}} \sum_{a=1}^A \sum_{b=1}^B w^c(a, b) \cdot (d^c(a, b) - m_{\theta}^c(a, b))^2 \quad (6)$$

in which w^c is a binary weighting matrix of size $A \times B$ taking into account the Bayer mask (w^c is equal to zero for 75% of the red and blue channels and for 50% of the green channel as shown in Figure 4) and any defective pixel on channel c . More generally, $w^c(a, b)$ stands for the inverse of the noise variance at pixel (a, b) on channel c [45]. The estimation process is performed jointly on the three channels ($c \in \{R = 1, G = 2, B = 3\}$).

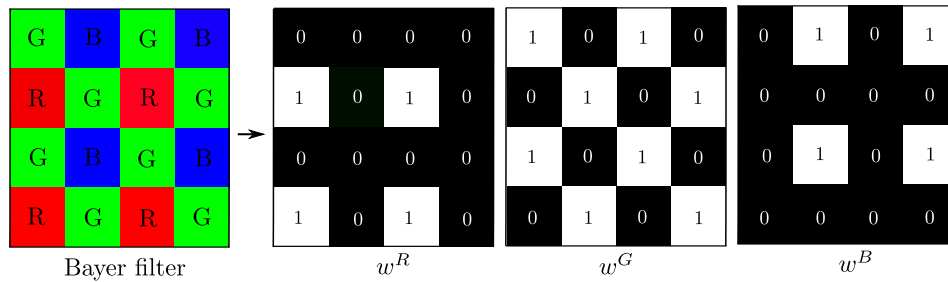


Fig. 4. Bayer filter and weighting matrices w^c for each color channel.

4. SELF-CALIBRATION METHOD FOR A LENSLESS COLOR MICROSCOPY SETUP

The self-calibration process described hereafter have several advantages. First of all, it can be considered as a cost-effective method as it only requires a spherical and calibrated opaque object. It is also simple to implement and fast. The overall process takes less than 4 minutes to obtain one estimate of wavelengths and 5 minutes to obtain one estimate of the crosstalk (on *Intel™ i7-4800MQ*, clock rate = 2.70 GHz) which seems reasonable in comparison with an experimental measurement of these parameters.

In order to study the proposed calibration method, we used a chrome deposited opaque object behaving as an opaque sphere. Its radius was $15\mu\text{m}$ (*Optimask™*, diameter $\pm 1\mu\text{m}$, roundness error $\pm 0.25\mu\text{m}$). It should be noted that, in practice, any spherical opaque object satisfying the Thompson model [44] can be used.

A. Estimation of the wavelengths of the sources

From equation 2, it is clear that it is impossible to jointly estimate the three wavelengths values as well as the z_p parameter (underdetermined system). It is thus necessary to select one of the wavelengths as a reference that will not be estimated. In practice, the laser source with the smallest uncertainty is chosen. As a result, in the following it is assumed that λ_G^{real} is equal to λ_G^{nom} , since the green laser wavelength used has the smallest manufacturer uncertainty (± 1 nm).

A.1. Principle of wavelengths estimation using the parametric inverse approach

The method to estimate the wavelengths can be broken down into four steps.

Step ①: Acquisition of three monochromatic holograms at the wavelengths λ^{real} that have to be estimated.

Step ②: Independent reconstruction of each hologram using the inverse parametric approach with nominal wavelength values λ^{nom} (no prior on the real wavelength values). Thus, at the end of this step, estimated parameters $\{x_{\{R,G,B\}}^{est}, y_{\{R,G,B\}}^{est}, z_{\{R,G,B\}}^{est}, r_{\{R,G,B\}}^{est}\}$ are obtained.

Step ③: First estimation of wavelengths of the sources. It can be deduced from the hypothesis that λ_G^{real} equals λ_G^{nom} that z_G^{est} and r_G^{est} are respectively the best estimation of z_p and r_p in the current step. From equations 1 and 2, a first estimated wavelength of the red and blue sources is given by:

$$\begin{cases} \lambda_R^{est1} &= \lambda_R^{nom} + \Delta\lambda_R = \lambda_R^{nom} + \lambda_R^{nom} \cdot \frac{z_G^{est} - z_R^{est}}{z_G^{est}} \\ \lambda_B^{est1} &= \lambda_B^{nom} + \Delta\lambda_B = \lambda_B^{nom} + \lambda_B^{nom} \cdot \frac{z_G^{est} - z_B^{est}}{z_G^{est}} \end{cases} \quad (7)$$

Step ④: Refinement of the red and blue wavelengths. This estimation is carried out using an inverse problem approach in which the parameters z_p and r_p are jointly estimated. The transversal coordinates (x_p, y_p) and the red and blue wavelengths are also estimated. The estimation process is initialized to the most accurate parameter values available at this step: $\{x_{\{R,G,B\}}^{est}, y_{\{R,G,B\}}^{est}, z_G^{est}, r_G^{est}\}$ and $(\lambda_R^{est1}, \lambda_G^{nom}, \lambda_B^{est1})$. At the end of this step, a second estimated wavelength $(\lambda_R^{est2}, \lambda_G^{nom}, \lambda_B^{est2})$ of the sources is obtained.

It is important to note that **Step ③** is a first estimate to be refined in **Step ④**. By initializing **Step ④** to estimated parameters closer to the real parameters than those available in **Step ②**, the computation time of **Step ④** is reduced. It also reduces the risk of converging to a local minimum of the cost function given in equation 6.

A.2. Experimental results and discussion

This method was applied to a set of 65 experimental holograms ($z_p \simeq 7$ cm, $r_p = 15$ μm transversely shifted by a few pixels from each other) and compared with measured wavelengths using a high-resolution spectrometer (*OceanOptics™*, QE65000) calibrated with a mercury spectral lamp (*Philips™*, 93136E). A second-order polynomial interpolation is applied to reduce measurement inaccuracy of the spectrometer from 0.8 nm to 0.3 nm on the central wavelength of the laser. Table 3 lists the mean value and standard deviation of estimated wavelengths $\{\lambda_R^{est2}, \lambda_B^{est2}\}$ (line 1 and 3) as well as the measured wavelengths (line 2). Line 1 gives the estimates if the green wavelength is set at its nominal value λ_G^{nom} (no prior on the wavelengths) and line 3 gives the estimates if the green wavelength is set at the wavelength λ_G^{mes} measured using a spectrometer.

Table 3. Estimated and measured wavelengths of the sources

	λ_R (nm)	λ_G (nm)	λ_B (nm)
Estimation ($\lambda_G^{ref} = \lambda_G^{nom}$)	638.3 \pm 0.2	532(ref.)	403.3 \pm 0.2
High-resolution spectrometer	637.8 \pm 0.3	531.4 \pm 0.3	402.9 \pm 0.3
Estimation ($\lambda_G^{ref} = \lambda_G^{mes}$)	637.6 \pm 0.2	531.4(ref.)	402.8 \pm 0.2

Within our experimental framework, it appears that the hypothesis assimilating the real green wavelength to its nominal value result in overestimation of 0.6 nm. By considering the equation 1 and a unique z for the three wavelengths, this bias generates a theoretical overvaluation of 0.7 nm for the red wavelength as well as a theoretical overvaluation of 0.5 nm for the blue wavelength. This is in agreement with the measured wavelengths of the sources obtained experimentally.

This method is constrained by the hypothesis assuming one of the three wavelengths to its nominal value. The uncertainty on the estimated wavelengths (for the two other sources) will increase with an increase in the manufacturer's uncertainty range on the

wavelength chosen as reference. In this experiment, this hypothesis generates an uncertainty of ± 1 nm on the wavelength value of this source. This leads to an uncertainty of ± 1.2 nm on the estimated red wavelength λ_R^{est2} and an uncertainty of ± 0.8 nm on the estimated blue wavelength λ_B^{est2} . Thus, using this estimation process reduces the uncertainty on the range of wavelength from 15 nm to $1.2 \times 2 = 2.4$ nm for the red source. Similarly, the uncertainty on the range of wavelength has been reduced from 10 nm to $0.8 \times 2 = 1.6$ nm for the blue source. It is thus shown by Table 3 line 3, that the absence of error in the hypothesis assimilating λ_G^{real} to λ_G^{nom} leads to a very accurate estimation of the two other wavelengths.

Moreover, beyond the relative precision of the estimation, it should also be noted that the deviations between the different estimated wavelengths are equal to the deviations between the different measured wavelengths. This point again emphasizes the interest of this method of estimation to improve the quality of the reconstructions. The limit of the method is reached when the uncertainties on the range of wavelength associated to each source provided by manufacturer are similar. In that case there is no gain in applying this method.

The proposed method is limited to estimating the central wavelength of the sources, while in practice the sources are not monochromatic ones. In our setup, according to *Thorlabs*TM CPS635R and CPS405 datasheets, a temperature variation may also result in a slight change in the shape of the laser emission spectrum. In the literature, the effects of the spectral width of the light source on the hologram formation have been studied. It is thus shown that a non-monochromatic source generates a low-pass filtering of the propagation impulse response by a super-Gaussian function depending on the spectral width of the source [46, 47]. This low-pass filtering effect will be added to the low-pass filtering due to the pixel integration. It should be possible to estimate this global low-pass filtering. Nevertheless, it appears that the spectral width is less crucial than the central wavelength of the sources. Indeed, not taking into account the spectral width in the image formation model (equations 3 and 4) leads to low residuals ($d - m_\theta$) comparable to an additive white and Gaussian noise, proving a good agreement between the direct model we use and the experimental data.

B. Calibration of the crosstalk phenomenon affecting the sensor

As detailed in Section 2, the optical crosstalk which is spatially dependent can be neglected in lensless microscopy. In the following, we do not attempt to separate the contribution of the spectral crosstalk with respect to the electronic crosstalk, but rather obtain an estimate of the overall crosstalk that can be included in the hologram formation model in order to improve the quality of the holographic reconstructions.

As a result, the proposed self-calibration method consists in determining a unique set of coefficients modeling the mixture of the signals received on each channel due to both the spectral and the electronic crosstalks.

B.1. Principle of crosstalk estimation using the parametric inverse approach

A simple experimental method based on an inverse parametric approach is proposed to determine the mixing coefficients between the different color channels for a fixed wavelength set. In presence of crosstalk, equation 3, which models the intensity diffracted on channel c should be replaced by equation 8, which takes into account the mixture of signals coming from the different channels:

$$m_\theta^c(x, y) = m_0^{c*} - \sum_{\ell=1}^3 \alpha_{\lambda(\ell)}^c \cdot g_{\lambda(\ell), \theta}(x, y) \quad (8)$$

where $\alpha_{\lambda(\ell)}^c$ represents the multiplicative factor between the holographic data recorded on channel c and the model generated at the wavelength $\lambda(\ell)$, and m_0^{c*} is the offset that results from mixing offsets m_0^R , m_0^G and m_0^B .

The signal ratio $q_{\lambda(\ell)}^c$ produced by an illumination at wavelength $\lambda(\ell)$ and which is actually recorded on sensor channel c due to the spectral crosstalk effect can be estimated by the ratio given by:

$$q_{\lambda(\ell)}^c = \frac{\alpha_{\lambda(\ell)}^c}{\sum_{c \in \{R, G, B\}} \alpha_{\lambda(\ell)}^c} \Rightarrow Q = \begin{pmatrix} q_R^R & q_R^G & q_R^B \\ q_G^R & q_G^G & q_G^B \\ q_B^R & q_B^G & q_B^B \end{pmatrix} \quad (9)$$

Thus, the mixing matrix Q characterizes the crosstalk effect. Its nine mixing coefficients $q_{\lambda(\ell)}^c$ are obtained by recording the three color holograms produced by the illumination of a parametric object at the three available wavelengths. For each of these three holograms, reconstructions have to be carried out for each of the three hologram channels using a parametric model for each wavelength. The multiplicative factors $\alpha_{\lambda(\ell)}^c$ correspond to the correlation coefficient between data recorded on channel c illuminated by wavelength $\lambda(\ell)$ and the model at this same wavelength. The principle of the crosstalk calibration method is illustrated in Figure 5 for wavelength λ_R . A non-zero signal is visible on the three color channels, thereby proving the presence of crosstalk. As detailed in Section 2, the crosstalk phenomenon cannot be accurately estimated by an average of the intensities received on each channel due to the nonlinearity of the camera response and to the non-uniform background varying over time. The proposed calibration method takes advantage of the modulation of the recorded signal induced by the holographic object to accurately estimate this phenomenon.

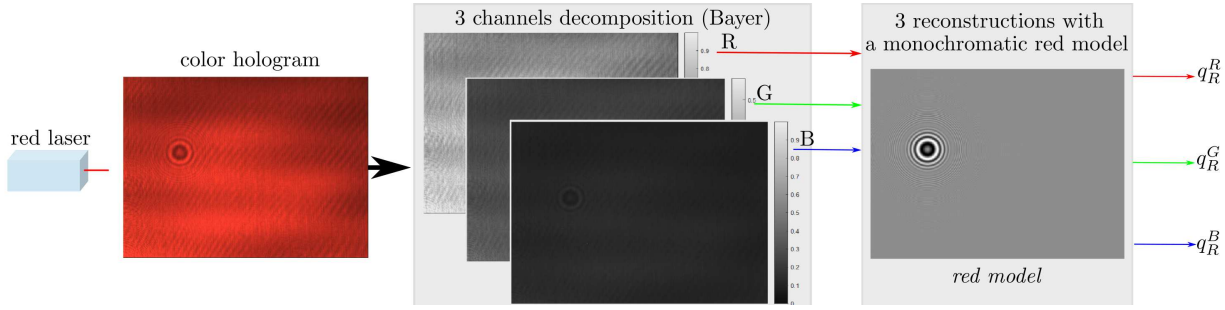


Fig. 5. Parametric estimation principle of channel mixing coefficients induced by the crosstalk phenomenon.

B.2. Experimental results and discussion

Table 4 gives the estimated mean and standard deviation (related to the standard deviation of the estimated parameters θ) of the crosstalk mixing matrix Q_{exp} obtained from 65 sets of three holograms recorded at the respective wavelengths $\{\lambda_R^{real}, \lambda_G^{real}, \lambda_B^{real}\}$. Table 5 gives the Q_m mixing matrix values extracted from the data provided by the manufacturer.

Table 4. Estimated crosstalk mixing matrix Q_{exp} (in percent)

	C^R	C^G	C^B
λ_R	74.9 ± 0.1	15.4 ± 0.1	9.7 ± 0.1
λ_G	11.7 ± 0.1	72.4 ± 0.2	15.9 ± 0.2
λ_B	14.0 ± 0.1	11.1 ± 0.1	74.9 ± 0.1

Table 5. Manufacturer's spectral crosstalk mixing matrix Q_m (in percent), deduced from Basler™, daA2500-14uc documentation

	C^R	C^G	C^B
λ_R	74	16	10
λ_G	12	70	18
λ_B	14	14	72

Using this scheme, it appears that estimated mixing coefficients are very close to those deduced from the manufacturer spectral responses of the Bayer filters. This indicates that the crosstalk effect of our camera is mainly due to the spectral crosstalk (the electronic crosstalk effect being negligible). Thus, it appears that for each channel, approximately 25% of the signal of interest is transferred to the two other channels due to the crosstalk effect. This observation suggests that it is important to take this phenomenon into account in the holographic model used during the reconstruction step. Note that, in practice, only one set of holograms is needed to obtain a spectral crosstalk estimation (whose standard deviation is given in Table 4). The application of this method to another camera of the same make led to a crosstalk matrix close to the one obtained here. Minor variations can be attributed to the camera's variability (e.g. homogeneity of the filters, local orientation of the microlenses, etc.). However, in the following paragraph, it will be emphasized that it is beneficial to estimate the mixing matrix Q more accurately than by just reading the values provided by the manufacturer. Indeed, a significant discrepancy was observed for the blue light intensity distribution on the different channels between the estimated values and the values derived from the manufacturer datasheets.

B.3. Accounting for crosstalk in the parametric model

Here we focus on accounting for the effects of spectral crosstalk in the holographic model used for hologram reconstruction (see section 3). For that purpose, the vector g containing the holographic models on the different channels c is replaced by g^* according to:

$$g^*(x, y) = \begin{pmatrix} g_{\lambda, \theta}^{*R}(x, y) \\ g_{\lambda, \theta}^{*G}(x, y) \\ g_{\lambda, \theta}^{*B}(x, y) \end{pmatrix} = Q^T \cdot \begin{pmatrix} g_{\lambda, \theta}(x, y) \\ g_{\lambda, \theta}(x, y) \\ g_{\lambda, \theta}(x, y) \end{pmatrix} = Q^T \cdot g(x, y) \quad (10)$$

in which Q^T stands for the transposed of matrix Q . This equation expresses the mixture of signals from the different sources on each channel c . Figure 6 illustrates this principle. Rather than separating the signals received on each sensor channel (by unmixing), this method mixes the hologram formation model in the same way as the recorded data.

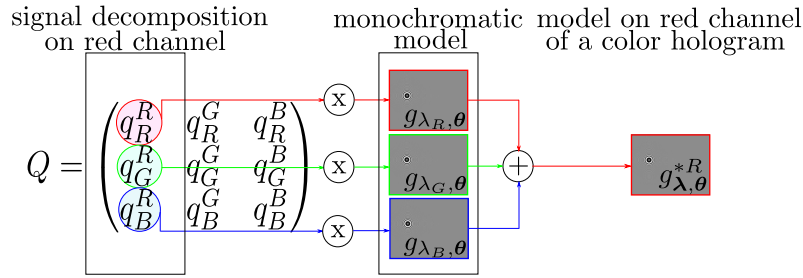


Fig. 6. Principle of the crosstalk correction.

We have applied the proposed crosstalk correction method on a set of 65 color holograms ($z_p \simeq 7$ cm, $r_p = 15$ μ m, with random transversal shifts at most a few pixels). Since the exact optical magnification of the setup is not precisely known, the absolute estimation bias cannot be obtained for real holograms. Thus, the quality of the reconstruction is evaluated based on the normalized correlation coefficient between the hologram formation model and the recorded data. We observed that taking the spectral crosstalk into account results on average in an improvement of the normalized correlation coefficient by 1.5%.

B.4. Influence of z_p and r_p on the quality of the reconstruction

In order to evaluate the influence of z_p and r_p on the quality of the reconstruction using the proposed crosstalk correction method, we have carried out numerical simulations of holograms for different sets of parameters z_p ($z_p \in [7\text{cm}; 12\text{cm}]$) and r_p ($r_p \in [15\mu\text{m}; 50\mu\text{m}]$). For each pair of values $\{z_p, r_p\}$, 100 color holograms (eq. 8) located in the center of the field of view are generated with an additive white and Gaussian noise and a signal to noise ratio (SNR) equal to 2.5.

Figure 7 shows the reconstruction bias on the estimated parameters z_p and r_p . Table 6 corresponds to one of the configurations illustrated in Figure 7.

Table 6. Bias and standard deviation on estimated parameters z_p and r_p without crosstalk correction (1st line), with crosstalk correction using Q_m (2nd line) and with crosstalk correction using Q_{exp} (3rd line) for a centered object whose radius is $r_p = 15\mu\text{m}$ located at $z_p = 7\text{cm}$ from the sensor plane

	bias on z_p (nm)	std. dev. on z_p (nm)	bias on r_p (nm)	std. dev. on r_p (nm)
without correction	52	316.3	21	5.0
with crosstalk correction (Q_m)	15	315.5	0.50	4.8
with crosstalk correction (Q_{exp})	9.9	315.5	0.28	4.8

Table 6 shows that taking the crosstalk into account in the hologram formation model reduces the estimation bias by a factor between 3 (using Q_m) and 5 (using Q_{exp}) for the estimated z_p parameter and by a factor between 20 (using Q_m) and 70 (using Q_{exp}) for the estimated r_p parameter for an object whose radius is $15\mu\text{m}$ and located at 7cm from the sensor plane. Correction by an approximate crosstalk matrix enables a significant reduction in estimation bias. This bias is further reduced when the crosstalk matrix used in the reconstruction step is closer to that generating the channels mixing. This justifies the use of an accurate estimation of spectral crosstalk. It should be noted that the standard deviation is not reduced by taking the spectral crosstalk into account. Indeed it is dependent on the SNR and on the estimation method. In addition, since a change of the z_p and r_p parameters causes a frequencies modification in the hologram signal, the crosstalk phenomenon can generate a more or less important frequency mixing according to the experimental parameters. As a result, the estimation of these parameters without crosstalk correction, generates a bias depending on the importance of the frequency mixing (especially important when z_p and r_p increase simultaneously, as shown by Figure 7). This shows that reconstructions without accounting for crosstalk cannot be easily compensated for the bias since the bias is not constant.

5. THEORETICAL LIMITS OF PRECISION

In this section, we discuss the theoretical limits of precision reachable on the estimated parameters $\theta = \{x_p, y_p, z_p, r_p\}$ in a monochromatic and color hologram recording framework. The aim is to investigate whether the consideration of a mosaicked color hologram which can be impacted by the crosstalk phenomenon, provides at least equal performance to those obtained with a monochromatic hologram.

The minimum achievable resolution on the estimated parameters can be statistically evaluated using the Cramér-Rao Lower Bounds (CRLBs). The variance of any unbiased estimator is bounded from below by the CRLBs, which are obtained from the diagonal of the inverse Fisher information matrix [48]. As a result, these limits represent the minimum estimate variance theoretically achievable when the noise is additive white and Gaussian. CRLBs constitute only a theoretical tool useful for relative comparison between different experimental configurations. By describing the light intensity recorded on the sensor by the direct model (equation 4) and by performing a statistical analysis on the estimated parameters for a given noise variance, it is possible to obtain the theoretical standard deviation δ_p for each estimated parameter $p \in \theta$. This method, which was applied to digital holography in [49], is applied here to

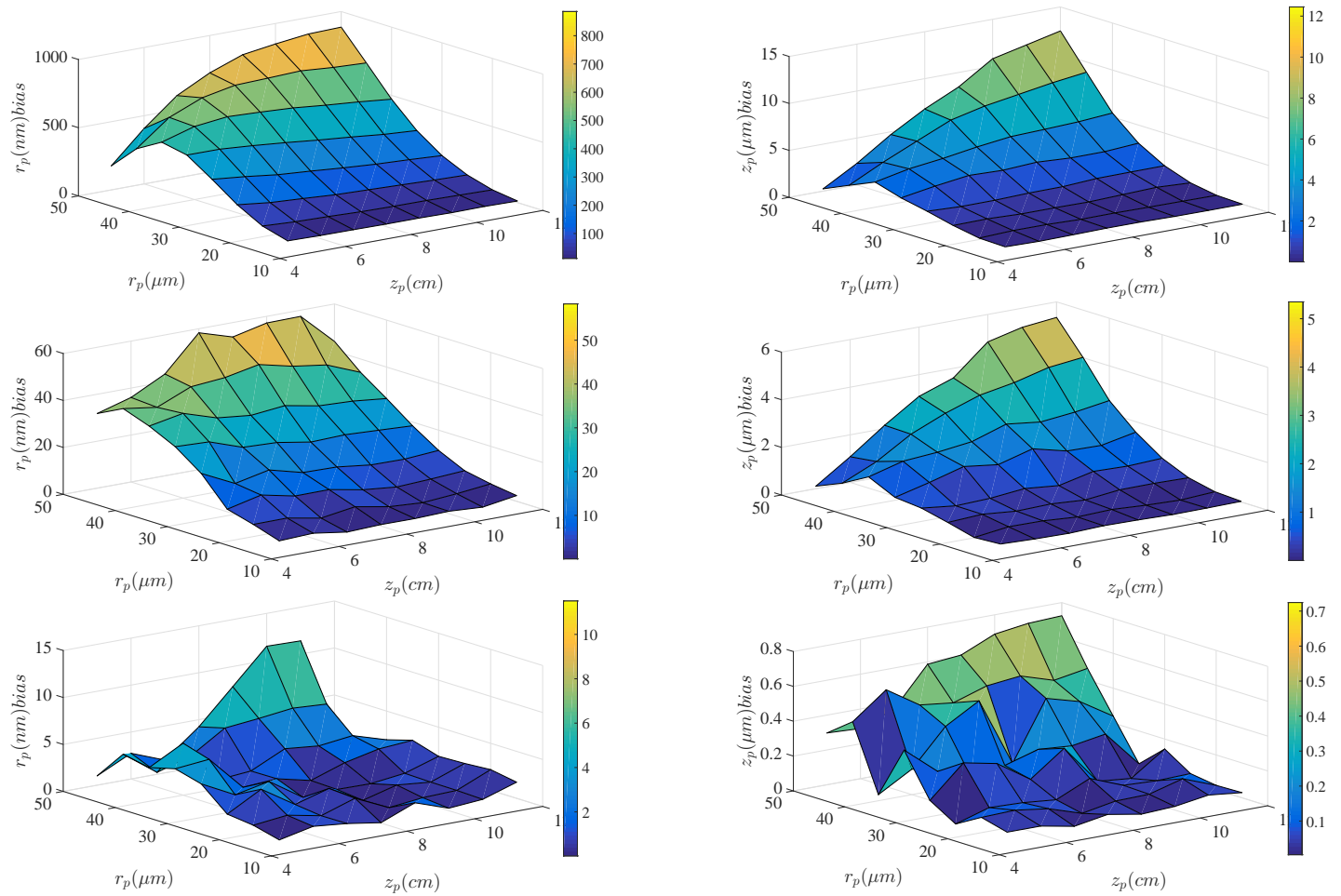


Fig. 7. Bias on estimated r_p (left) and z_p (right) without crosstalk correction (1st line), with crosstalk correction using Q_m (2nd line) and with crosstalk correction using Q_{exp} (3rd line) for a centered objet whose radius is $r_p \in [15\mu m; 50\mu m]$ located at $z_p \in [7cm; 12cm]$ from the sensor plane

color holograms in taking into account the possible mixture of the color signals due to a crosstalk phenomenon. Equation 11, gives the Fisher information matrix I depending on the model gradients, the amplitude factor of the model α and the variance σ_b^2 of the noise. Equation 12 gives the link between the CRLBs and the matrix I . When crosstalk signals mixing occurs, the color model vector g is replaced by g^* according to equation 10.

$$[I(\theta)]_{i,j} = \frac{\alpha^2}{\sigma_b^2} \sum_{a=1}^A \sum_{b=1}^B \left(\frac{\partial g_\theta(a,b) \cdot w(a,b)}{\partial \theta_i} \right) \cdot \left(\frac{\partial g_\theta(a,b) \cdot w(a,b)}{\partial \theta_j} \right) \quad (11)$$

$$\delta_i = \sqrt{[I(\theta)^{-1}]_{i,i}} \quad (12)$$

The theoretical limits of precision are obtained by considering a hologram model of a particle whose radius is $r_p = 15 \mu\text{m}$, located at the distance $z_p = 7 \text{ cm}$ with a SNR equal to 2.5. Table 7 gives the reachable theoretical resolutions according to each parameter $p \in \theta$. This study was performed in the six following cases:

- A monochromatic hologram obtained with only one source and recorded by a monochromatic sensor (cases 1 to 3, respectively denoted λ_R , λ_G and λ_B in table 7 and figure 8)
- A trichromatic hologram artificially formed by the concatenation of three holograms recorded by a monochromatic sensor without mosaicking as in [50, 51] (case 4 denoted $\lambda_{R,G,B}$)
- A mosaicked color hologram recorded by a Bayer color sensor not affected by the crosstalk phenomenon (case 5 denoted $\lambda_{R,G,B} + \text{Bayer}$)
- A mosaicked color hologram recorded by a Bayer color sensor affected by the crosstalk phenomenon defined by the mixing matrix Q_{exp} estimated from the experimental holograms (case 6 denoted $\lambda_{R,G,B} + \text{Bayer} + ct$).

The mean results may vary slightly depending on the position of the particle in the sensor field. Figure 8 completes this study by showing the evolution of the achievable precision limits for a distance between the sensor and the object varying from 7 to 12 cm in the same cases as those listed Table 7.

Table 7. CRLBs for the reconstructed parameters

	δ_x (nm)	δ_y (nm)	δ_z (nm)	δ_r (nm)
λ_R	0.56	0.49	31	0.37
λ_G	0.59	0.55	35	0.40
λ_B	0.70	0.63	41	0.46
$\lambda_{R,G,B}$	0.32	0.35	20	0.13
$\lambda_{R,G,B} + \text{Bayer}$	0.53	0.50	31	0.22
$\lambda_{R,G,B} + \text{Bayer} + ct$	0.70	0.80	40	0.30

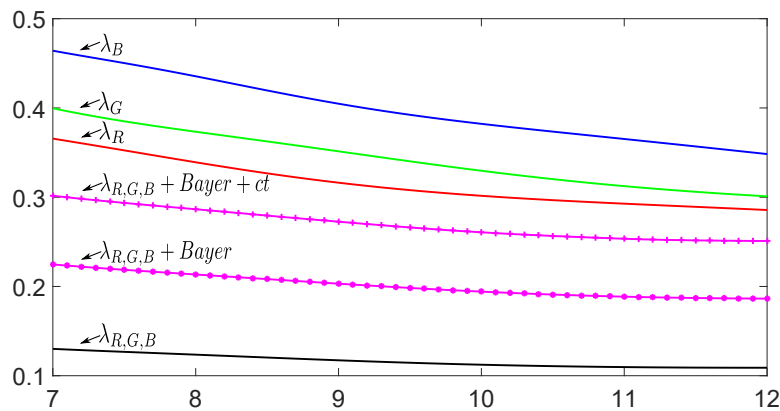


Fig. 8. CRLBs on the particle radius for different distances between the sensor and the object.

It appears that color holograms that undergo mosaicking during the acquisition step suffer from an increase in CRLBs (and hence a degradation in the reconstruction) with respect to a color hologram artificially formed by the concatenation of three monochromatic

holograms (obtained with the three available wavelengths in our experimental setup). This is explained by the fact that the mosaicking step results in a reduction by a factor of 3 in the number of pixels recorded, resulting in lost information. It also appears that holograms not tainted by a crosstalk phenomenon lead to finer resolutions than holograms tainted by this effect. This can be explained by the blurring of high frequencies due to mixing of the color signals when spectral crosstalk occurs. This observation justifies why the spectral crosstalk phenomenon that occurs with most color sensors equipped with a Bayer filter has an influence in metrological application using an in-line holographic setup. Finally, we observed that it is possible to obtain a finer precision of the r_p parameter by considering a color hologram, due to additional information provided by wavelength diversity.

6. CONCLUSION

In lensless color microscopy, calibration of the wavelength of the sources and of the crosstalk phenomenon caused by Bayer filters is important for quantitative imaging. We showed that inaccurate estimation of wavelengths or crosstalk leads to inaccurate reconstructions. We proposed an experimental self-calibration process based on an inverse problems reconstruction and studied its performance. This self-calibration is simple to apply since it only requires the lensless color microscopy setup. Any spherical object whose diffraction pattern can be described by a direct model (Thompson model [44] for opaque object or (Generalized) Lorenz-Mie model [52, 53] for droplets, bubble or beads) can be used. For estimation of laser diode wavelengths, the suggested method makes it possible to accurately estimate two wavelengths based on the value of the third taken as a reference. Considering the wavelength with the smallest uncertainty range given by the manufacturer as a reference, we demonstrate that it is possible to obtain uncertainty ranges for the two other wavelengths that are narrower than the typical ranges provided by the manufacturer. The uncertainty on the estimated wavelengths is reduced considerably if the uncertainty on the reference wavelength is small. Work is now underway to jointly estimate the three wavelengths using a set of several color holograms recorded at different distances between the sensor and the object, and to also estimate the widths of the spectral sources by taking them into account in the hologram formation model.

In the same way, we also proposed an experimental method using an inverse parametric approach to estimate the spectral crosstalk phenomenon that occurs on a color sensor equipped with a Bayer filter and which results in mixing of the channels. It has been shown that after having estimated this phenomenon it is possible to take it into account to reduce estimation bias or to increase the correlation between the recorded data and the hologram formation model.

Finally, it has been shown that the color holograms acquired with a Bayer color sensor suffer from a degradation of the theoretical limits of estimation precision with respect to a trichromatic color image that does not undergo mosaicking. Similarly, the presence of crosstalk leads to an additional degradation of the theoretical limits of precision due to the blurring introduced by mixing different color signals. However, the theoretical limit on the estimated radius of the holographic object is improved with respect to a monochromatic acquisition thanks to wavelength diversity. The methods for estimating wavelengths and crosstalk lead to calibration parameters that can be used whatever the final reconstruction method is.

7. FUNDING INFORMATION

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